Rotating and Accelerating Frames

Non-inertial Reference Frames Pivoted Rod, Truck Door, Rotating Pendulum, and Cylinder on Plank

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Before attempting the problems, please review the following background material:

- **Non-inertial Reference Frames:** In an accelerating frame, fictitious forces appear: $F_{\text{fictitious}} = -m\vec{a}_{\text{frame}}$
- **Rotating Frames:** Centrifugal force $F_{\text{cf}} = m\Omega^2 r$ (outward) and Coriolis force $F_{\text{Cor}} = -2m\vec{\Omega}\times\vec{v}$
- **Rigid Body Rotation:** Torque $\tau = I\alpha$, moment of inertia for a rod about end: $I = \frac{1}{3}ML^2$, for a cylinder: $I = \frac{1}{2}MR^2$
- **Rolling Without Slipping:** $v = \omega R$, $a = \alpha R$
- **Small Oscillations:** For small angles, $\sin\theta \approx \theta$, $\cos\theta \approx 1$

No calculus is required unless explicitly stated. Students are encouraged to rely on:

- Free-body diagrams in both inertial and non-inertial frames
- Effective gravity concepts
- Energy methods where applicable
- Constraint relationships for rolling motion

Conceptual Background

These problems explore the behavior of objects in accelerating and rotating reference frames. When analyzing motion from the perspective of a non-inertial frame, we must introduce fictitious forces to apply Newton's laws. For an accelerating frame (like a car or truck), we add a pseudo-force $-m\vec{A}$ opposite the acceleration. For a rotating frame, we add centrifugal and Coriolis forces. The equilibrium positions and oscillation frequencies are modified by these fictitious forces.

The cylinder on an accelerating plank combines rigid body dynamics with rolling constraints, requiring careful analysis of forces and torques in either the inertial or non-inertial frame.

Key Definitions / Laws

Torque equation (inertial frame):

$$\sum \vec{\tau} = I\vec{\alpha}$$

In an accelerating frame (acceleration $\vec{A}$):

$$\sum \vec{F} - m\vec{A} = m\vec{a}' \quad \text{(where } \vec{a}' \text{ is acceleration relative to frame)}$$

In a rotating frame (angular velocity $\vec{\Omega}$):

$$\sum \vec{F} - 2m\vec{\Omega} \times \vec{v}' - m\vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = m\vec{a}'$$

Rolling without slipping condition:

$$v_{\text{cm}} = \omega R, \quad a_{\text{cm}} = \alpha R$$

Moment of inertia for a uniform rod about one end:

$$I_{\text{rod}} = \frac{1}{3}ML^2$$

Moment of inertia for a solid cylinder about its axis:

$$I_{\text{cylinder}} = \frac{1}{2}MR^2$$

Problem 1. *Pivoted Rod on Car.*

A uniform thin rod of length L and mass M is pivoted at one end. The pivot is attached to the top of a car accelerating at rate A, as shown.

(a) What is the equilibrium value of the angle θ between the rod and the top of the car?

(b) Suppose that the rod is displaced a small angle ϕ from equilibrium. What is its motion for small ϕ?

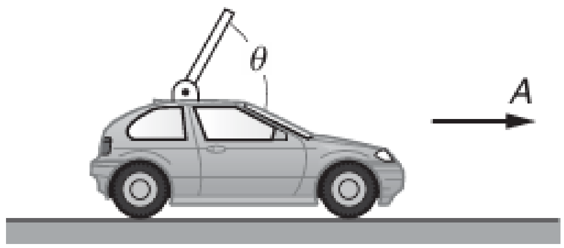

Figure 1: Pivoted rod on an accelerating car. The rod makes an angle θ with the horizontal.

Problem 2. *Truck Door.*

A truck at rest has one door fully open, as shown. The truck accelerates forward at constant rate A, and the door begins to swing shut. The door is uniform and solid, has total mass M, height h, and width w. Neglect air resistance.

(a) Find the instantaneous angular velocity of the door about its hinges when it has swung through 90°.

(b) Find the horizontal force on the door when it has swung through 90°.

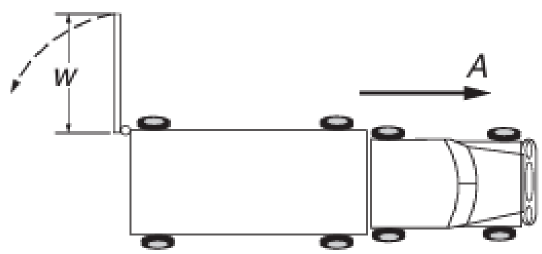

Figure 2: Truck door swinging shut as the truck accelerates forward. The door has width w and height h.

Problem 3. *Cylinder on an Accelerating Plank.*

A cylinder of mass M and radius R rolls without slipping on a plank that is accelerated at rate A. Find the acceleration of the cylinder.

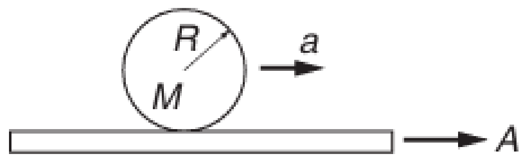

Figure 3: Cylinder rolling without slipping on a plank that accelerates at rate A.

Problem 4. *Pendulum on Rotating Platform.*

A pendulum is rigidly fixed to an axle held by two supports so that it can swing only in a plane perpendicular to the axle. The pendulum consists of a mass M attached to a massless rod of length l. The supports are mounted on a platform that rotates with constant angular velocity Ω. Find the pendulum's frequency assuming that the amplitude is small.

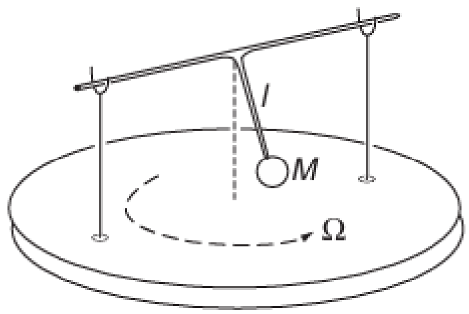

Figure 4: Pendulum on a rotating platform. The axle allows swinging only in a plane perpendicular to the rotation axis.

Topics covered before this problem set

1. Momentum Conservation
2. Projectile Motion
3. Statics
4. Rolling without slipping constraints

Topics covered in this problem set

1. Non-inertial reference frames (accelerating and rotating)
2. Fictitious forces (inertial forces)
3. Rolling without slipping on accelerating surfaces
4. Effect of rotation on pendulum frequency
5. Constraint equations for rolling motion

Next up

1. Angular Momentum
2. Advanced rolling problems (with slipping)